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PROFESSOR JOHN DYER MEMORIAL LECTURE
THE ORIGIN OF THE UNIVERSE FROM QUANTUM CHAOS
AN INTRODUCTION TO CURRENT IDEAS

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In his recently published book "A Brief History of Time", S. Hawking describes his remarkable insights into the problem of the origin of our universe. In this talk a more quantitative description of some of the important principles from this book is presented as a mathematical appendix to it. A brief review of the ideas of the Standard Big Bang Model of the Universe is given in terms of the evolution equation that follows from Einstein's theory. The meaning of the Cosmological Constant, its relation to Vacuum Energy, the model of the empty DeSitter Space and Gravity is derived. By analogy to Schrodinger mechanics one can give the general features of "Quantum Cosmology", in which the origin of the universe can be viewed as a Quantum tunneling process in imaginary time from a Quantum Chaos state of no space, no time, no matter to an inflationary expanding DeSitter space which eventually transits into the Hot Big Bang Expansion that we see.

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SUMMARY

In his recently published book "A Brief History of Time", S. Hawking describes his remarkable insights into the problem of the origin of our universe.

In this talk a more quantitative description of some of the important principles from this book is presented as a mathematical appendix to it. A brief review of the ideas of the Standard Big Bang Model of the Universe is given in terms of the evolution equation that follows from Einstein's theory. The meaning of the Cosmological Constant, its relation to Vacuum Energy, the model of the empty DeSitter Space and the Inflationary Model are introduced. Then the Uncertainty Principle for Quantum Gravity is derived. By analogy to Schrodinger mechanics one can give the general features of "*Quantum Cosmology*", *in which the origin of the universe can be viewed* as a Quantum tunneling process in imaginary time from a Quantum Chaos state of no space, no time, no matter to an inflationary expanding DeSitter space which eventually transits into the Hot Big Bang Expansion that we see.

Dedication:

This talk was presented at the Colloquium of the Physics Department at the Naval Postgraduate School to honor the memory of Professor John N. Dyer who died suddenly and unexpectedly on 31 December 1988. John's dedication to teaching and to the mission of this institution has been an example to all who had the privilege of working closely with him. His curiosity about new concepts in physics and his ability to make them accessible to a wider audience, focusing on the fundamental ideas without getting lost in technical details and yet preserving the depth of the arguments have been an inspiration to those of us who worked with him. The clarity and elegance of his class notes was the envy of his colleagues and the delight of his students. John will be sorely missed and long remembered by all of us.

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THE ORIGIN OF THE UNIVERSE FROM QUANTUM CHAOS

1. INTRODUCTION

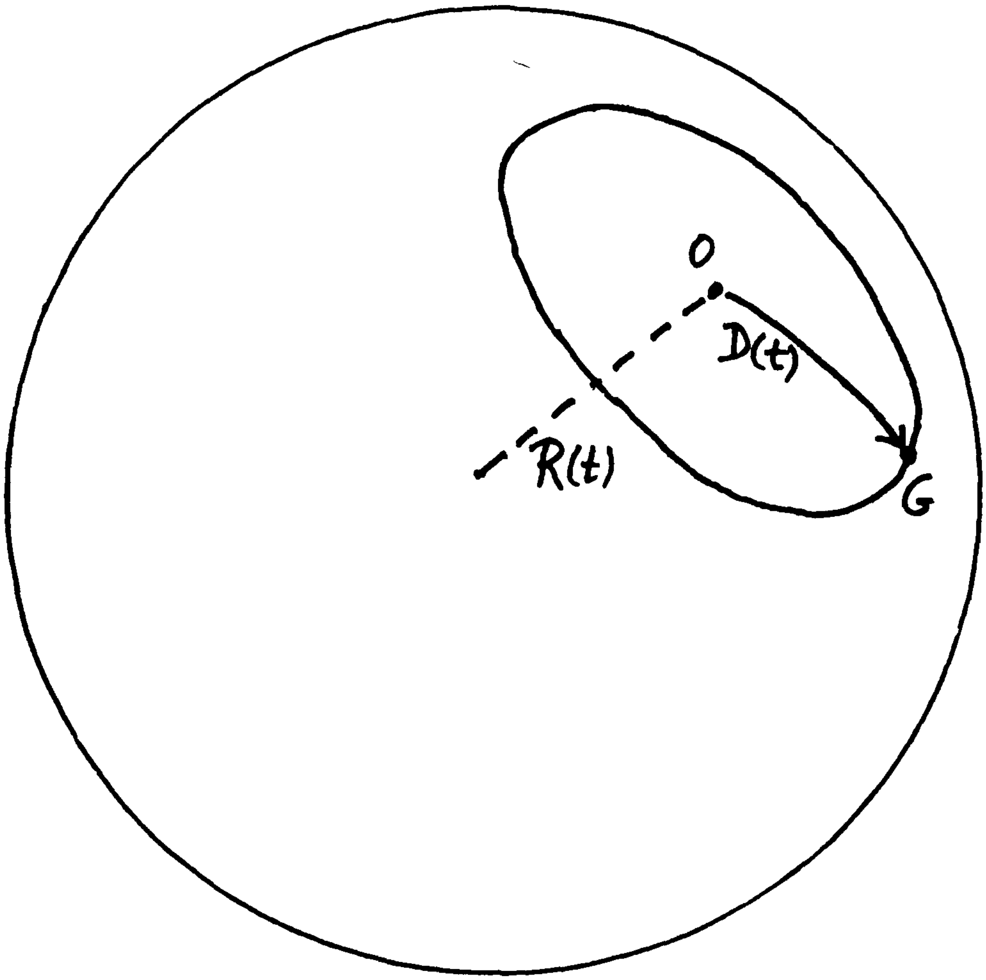
In his recently published book "A Brief History of Time", Stephen W. Hawking, (1) one of the most remarkable thinkers of our time, described his inquiry and his insights into the question of the origin of the universe, the origin of space, time and matter. The book is written for a wide audience without the use of mathematical formalism.

In this talk I would like to supply, so to speak a mathematical appendix to this book which, I believe, will actually help you to understand the more complicated concepts at the end of the book, dealing with Quantum Cosmology. After a brief review of the ideas of the standard Big Bang Model of the Universe we will introduce the important concepts of Vacuum Energy, the empty DeSitter Space and the Inflationary Scenario. Then we present the Uncertainty Principle for Quantum Gravity. By analogy to Schrodinger mechanics one can give the general features of a "Quantum Cosmology", in which the origin of the universe can be viewed as a Quantum tunneling process in imaginary time from a Quantum Chaos state of no space, no time, no matter to an inflationary expanding DeSitter space which eventually transits into the Hot Big Bang Expansion that we see. This exposition is not meant as a report about a actual model but rather as an introduction to the kind of principles that are now under discussion in the literature.

2. THE STANDARD BIG BANG UNIVERSE

The confluence of Einstein's theory of Gravitation, the General Theory of Relativity and the astronomical observations made by Edwin Hubble around 1930, which showed that distant galaxies appear to recede from us with speeds propertional to their distance has led to the modern form of theoretical cosmology. The widely accepted model is that the space of our universe is not a static absolute arena *into which the material objects are placed.* Rather, space itself, not necessarily euclidean but curved, is undergoing evolution. Space is expanding and with this expansion the objects in this space are receding from each other. We can best visualize this concept by assuming that space is 2–dimensional rather than 3–dimensional and is represented by the surface of a balloon (Fig.1). All physical processes in this 2–dimensional universe are entirely confined to the surface of the balloon. Masses are dots on this surface and light signals are ripples that propagate with speed c along the curved surface. As the radius of the balloon increases at a certain rate any two points on the surface separated by a distance D at time t will recede from each other with a rate $\dot{D}$ which is proportional to D by simple geometric proportion shown in this picture. In 3 dimensions, this is precisely the observed "Hubble Effect". The dynamics of this expansion process is governed by Einstein's general theory of relativity. In the simple case of a homogeneous isotropic universe filled with matter and radiation given by the present mass ensity ρ_0 the dynamics follows an energy conservation law (Fig.2). The first term is the normalized square of the rate of expansion and is a kinetic energy term. The second term is the gravitational potential energy due to the mutual attraction of all the masses. On the right hand side is the total mass energy of the system. Einstein had originally introduced another force into his theory in addition to the attractive gravitational force. This cosmological force was repulsive and was needed to balance the gravitational force so as to produce a static universe. Prior to the discovery of the Hubble Effect it was the prevailing belief that the universe is static. When the general Hubble expansion was discovered, this

GEOMETRY OF THE EXPANDING EINSTEIN UNIVERSE



$$\frac{\dot{D}}{D} = \frac{\dot{R}}{R} \equiv H \quad \text{HUBBLE CONSTANT}$$

$$\dot{D} = \begin{array}{c}\text{velocity of recession}\\ \text{of galaxy } G\end{array} = \left(\frac{\dot{R}}{R}\right) D$$

FIG. 1

1 COSMOLOGICAL MODELS WITH COSMOLOGICAL CONSTANT AND RADIATION DOMINANCE

$$\boxed{\underbrace{\left(\frac{\dot{R}}{R_0}\right)^2}_{\text{Kinetic Energy}} - \underbrace{\frac{\rho_0}{\rho_c} H_0^2 \left(\frac{R_0}{R}\right)^2}_{\text{Gravitational Potential Energy}} = \underbrace{-H_0^2\left(\frac{\rho_0}{\rho_c}-1\right)}_{\text{Total Mass-Energy}} + \underbrace{\frac{1}{3}\lambda\cdot\left(\frac{R}{R_0}\right)^2}_{\text{Cosmological Vacuum Energy}}}$$

$H_0 \equiv \frac{\dot{R}_0}{R_0}$ Measured Hubble Constant $\rho_c \equiv \frac{3}{8\pi G} H_0^2$ critical Mass density

$$\lambda \lll H_0^2$$

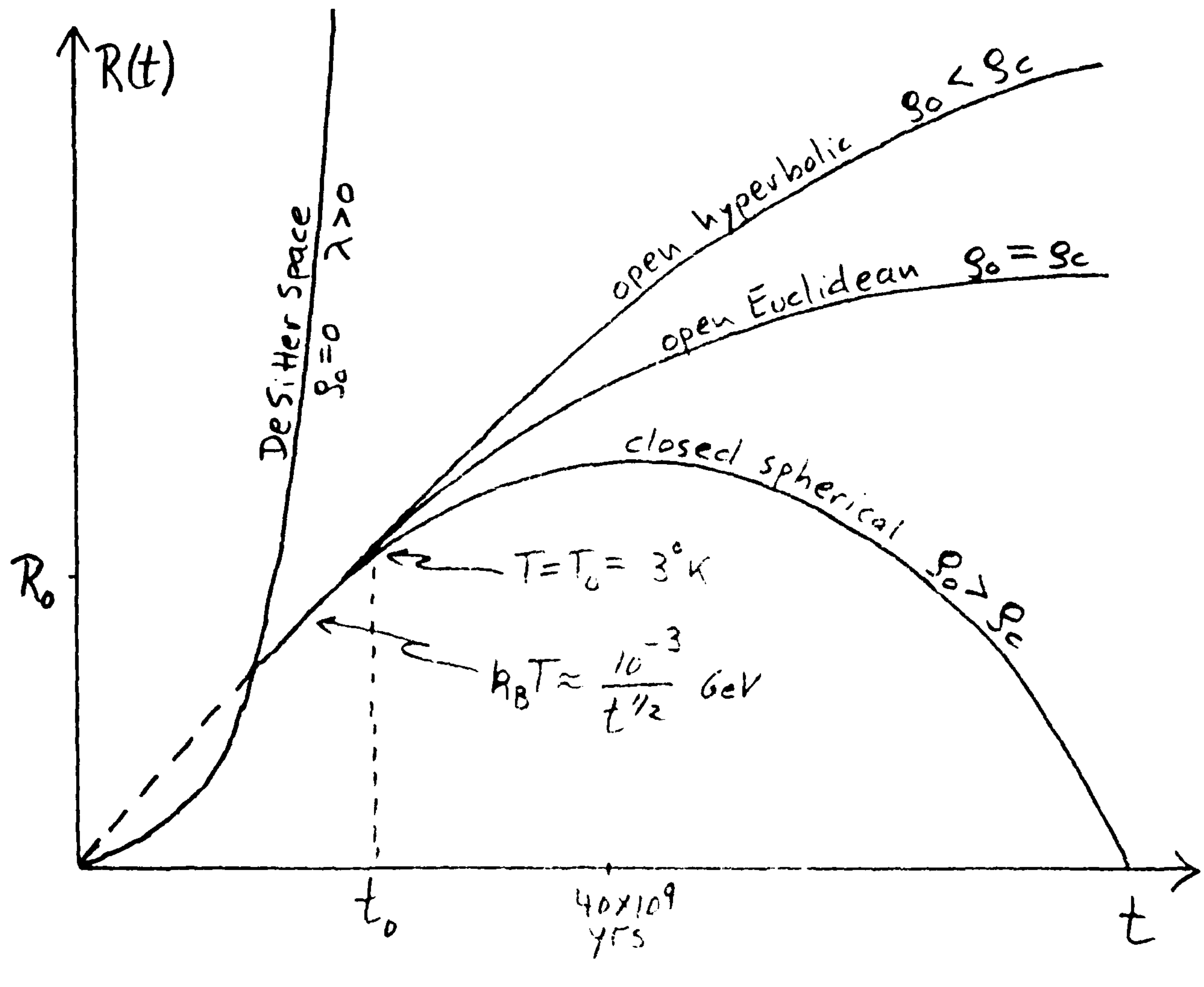


FIG. 2

artifact of a cosmological repulsive force was no longer needed and was deleted. Today this term has been reinstated with high honors and plays a major role in the story, as we will see shortly. The density ρ_c is a critical density. All solutions of this differential equation, the energy equation, have a "singularity" at t=0 where the "radius" R of the universe is zero and the density is infinite. Due to the attractive forces between all the masses the expansion is decelerating. If the present mass density ρ_0 is larger than the critical density then the total energy is negative, the attractive forces eventually eat away all the kinetic energy and just like a rock projected upward in the earth gravitational field with less than escape velocity the masses will return on themselves, the universe will recollapse into a singularity of zero radius and infinite density. If the present density is less than critical then there is not enough overall attraction, the system has escape velocity, the total energy is positive, the universe expands forever. At the present time t_o, 20 billion years after the "Big Bang", the temperature in this universe as given by the temperature of the so called Black–Body Background Radiation (discovered in 1964),is 3^oK. The temperature changes with the expansion time like $t^{-\frac{1}{2}}$ and thus was much higher in earlier epochs of the universe.

3. THE ROLE OF THE COSMOLOGICAL CONSTANT – DeSITTER SPACE

In the discussion of the Standard Cosmological Model of the previous section the cosmological term was neglected. Indeed, from astronomical observations we know that λ must be very small, perhaps zero. But let us explore the deeper meaning of that cosmological term.In the energy equation it represents an additional positive energy term that adds to the total energy and is proportional to the size of the universe. One obtains some insight into the peculiar nature of this energy by returning to the dynamic equations that follow from general Relativity (Fig.3). We see here two equations (written in the non–normalized form) (a) is the energy equation again, (b) an equation of motion that

describes the deceleration due to the gravitational attraction of the masses of density ρ and pressure p and the repulsive cosmological force proportional to R. By simply rewriting and including the cosmological term with the mass density term one discovers that the equations with the λ–term behave exactly like the equations without λ–term but a density increased by $\lambda/_{8\pi G}$. The quantity $\lambda/_{8\pi G}$ acts like a mass density in the universe. It represents a mass–energy density

$$\rho_{vac}c^2 = \frac{\lambda c^2}{8 \pi G}$$

which is present even if the universe contains no real mass. Thus energy is present even in a vacuum space. Peculiarly enough, the pressure of this "vacuum substance" is negative and is of the same magnitude as the energy density $\rho_{vac}c^2$. The situation can be compared with a vapor bubble in a hot liquid (Fig. 4). The attractive gravitational force which tries to contract the balloon surface is like the surface tension of the liquid bubble surface. The repulsive cosmological force which tries to increase the balloon surface is like the vapor pressure inside the bubble. This vapor pressure represents energy in the system that can do work of expansion. This energy is present even if the total mass on the balloon surface is negligible. The larger the vapor bubble becomes the more of the liquid is being converted to vapor, adding the latent heat of vaporization to the bubble system. Some of that energy is used to overcome the surface tension. From the point of view of the inhabitants of the balloon surface energy is used up to do work against the surface tension. That implies that an increase of the volume corresponds to a decrease of cosmological energy. Therefore the corresponding pressure must be considered negative

$$dU_{cos} = P_{cos} \cdot dV < 0$$

EINSTEIN THEORY WITH COSMOLOGICAL CONSTANT

(a) $$\dot{R}^2 - \frac{8\pi G}{3}\rho R^2 - \frac{1}{3}\lambda R^2 = -c^2 k$$ Energy equation

(b) $$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)R + \frac{1}{3}\lambda R$$ Momentum Equation

$\Downarrow$ rewrite

$$\dot{R}^2 - \frac{8\pi G}{3} R^2\left(\rho + \frac{\lambda}{8\pi G}\right) = -c^2 k$$

$$\ddot{R} = -\frac{4\pi G}{3} R\left[\left(\rho + \frac{\lambda}{8\pi G}\right) + \frac{3}{c^2}\left(P - \frac{\lambda c^2}{8\pi G}\right)\right]$$

$\hookrightarrow$ $\frac{\lambda}{8\pi G}$ acts like a density which is present even in the vacuum state with $\rho = 0$

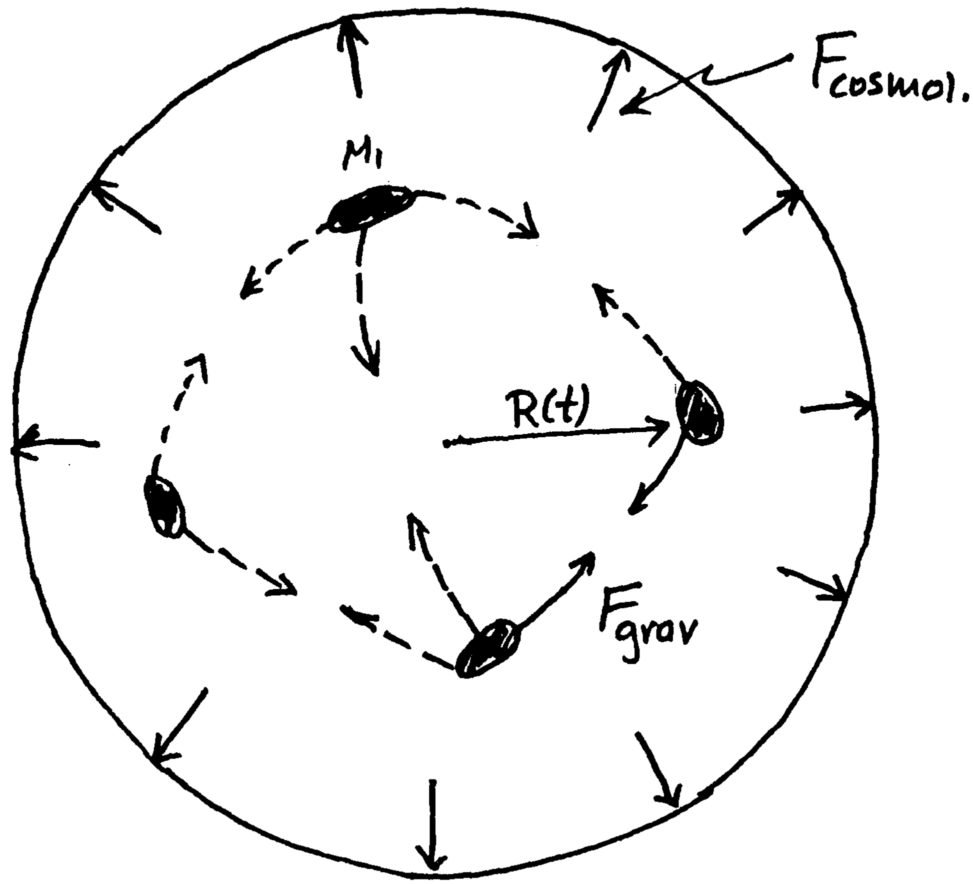
$$\rho_{vac} c^2 \equiv \frac{\lambda c^2}{8\pi G}$$ "Vacuum Energy Density"

$$P_{vac} = -\frac{\lambda c^2}{8\pi G} = -\rho_{vac} c^2$$ "Negative Vacuum Pressure"

FIG. 3

DYNAMICS OF EXPANDING UNIVERSE WITH GRAVITATIONAL AND COSMOLOGICAL TERM

Balloon Model — Vapor Bubble in hot liquid.



gravitational Attraction — Liquid Surface Tension

Cosmological Repulsion — Inside Vapor pressure

FIG. 4

At this point the vacuum energy density

$$\rho_{vac}c^2 = \frac{\lambda c^2}{8 \pi G}$$

is still an ad hoc speculative concept which is not connected to anything in our laboratory experience of physics. We will make that connection in the next section. A very interesting consequence of the cosmological term is the existence of cosmological models with zero mass content, $\rho \equiv 0$ (Fig. 5). If the parameter k is set to zero, a particularly simple model emerges, a exponentially expanding space with a exponential growth rate

$$\tau^{-1} = \sqrt{\lambda/3} = \sqrt{\frac{8 \pi G}{3 c^2} \rho_v c^2} \, .$$

Such a space is called a DeSitter space. Only seven years ago the DeSitter space solution became of practical interest to cosmologists in connection with certain problems of the standard model and their solution in the so called inflationary scenario.

4. THE INFLATIONARY UNIVERSE SCENARIO

There were a number of problems plaguing the standard model. One was the observation that the observed mass–energy density ρ_0 in the universe is very close to the critical density so that the total energy is close to zero which would require an extremely accurate initial condition for the Big Bang. The second is the observation that the cosmic background radiation arriving today from opposite directions in the universe is isotropic to a very high degree. Yet when this primordial radiation was emitted in the early stages of the universe the sources at opposite sides of the universe from us could not have been in thermodynamic equilibrium. This is due to the fact that in an expanding universe each

THE EINSTEIN – DESITTER UNIVERSE

$$\dot{R}^2 - \frac{8\pi G}{3}\rho R^2 - \frac{1}{3}\lambda R^2 = -c^2 k$$

Assume: Universe is empty $\rho = 0$
Universe is "flat" $k = 0$

$$\dot{R}^2 = \frac{1}{3}\lambda R^2$$

Solution:

$$R(t) = R_i e^{\sqrt{\frac{1}{3}\lambda}(t - t_i)}$$

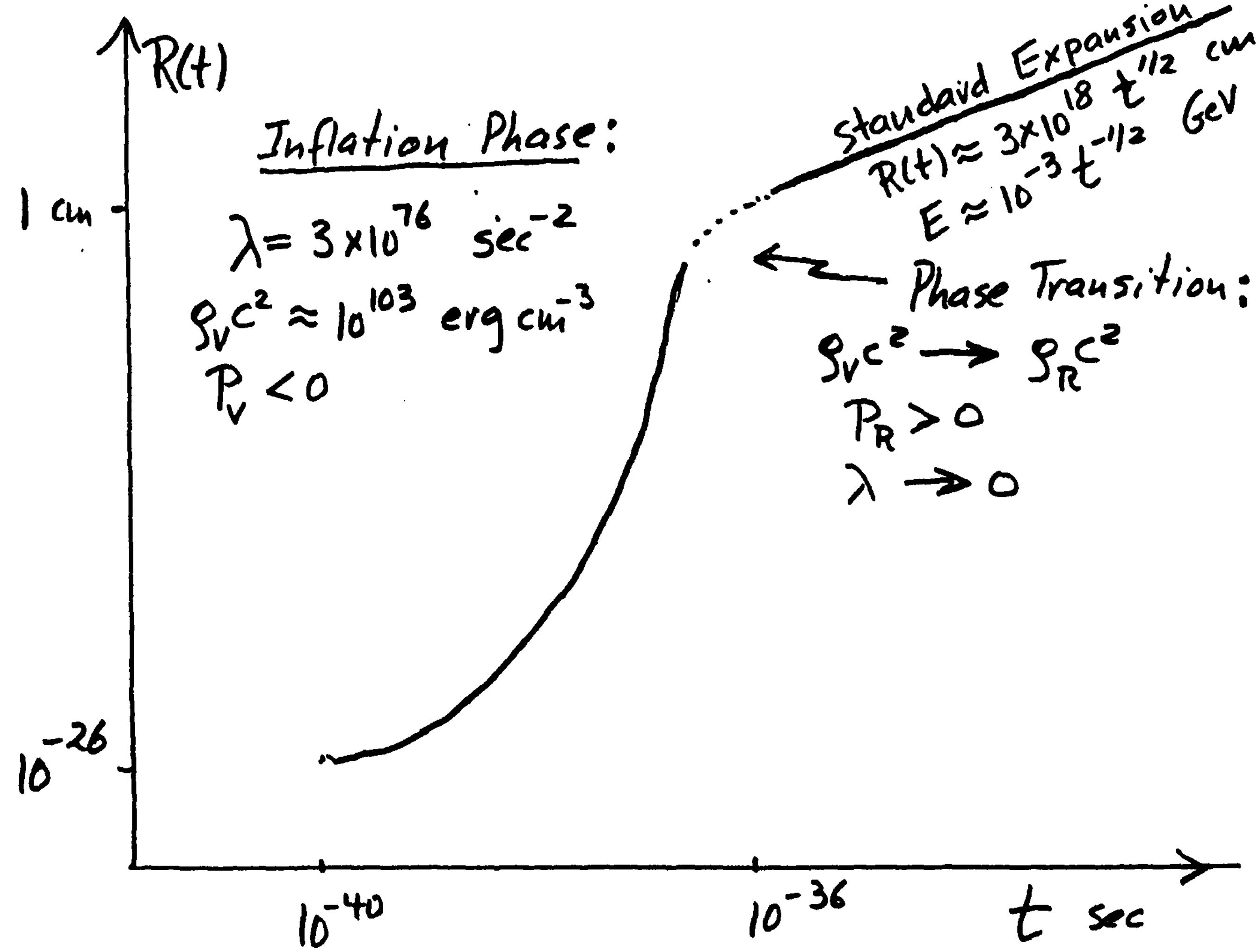
Vacuum Energy density: $\rho_v c^2 \equiv \dfrac{\lambda c^2}{8\pi G}$

FIG. 5

observer has an absolute horizon such that radiation from sources beyond that horizon would take more time to reach the observer than is available since the Big Bang. In 1981 A.Guth proposed a solution to these problems by hypothesizing that in the very early epochs of the universe, prior to 10^{-36} sec after its appearance the universe underwent a dramatic initial inflationary expansion that increased the radius in less than 10^{-36} seconds by more than 26 orders of magnitude (Fig. 6). This exponential expansion was driven by a large cosmological constant representing a large vacuum energy of the order 10^{103}ergcm^{-3}, say. Otherwise this expanding Desitter Space would be without any physical mass energy. The energy we see today was only present in the latent form of the very large vacuum energy density. At about 10^{-36} sec some kind of phase transition of space occurs in which the latent energy "condenses out" and becomes physically measurable energy in exchange for the vacuum energy. The formerly very large cosmological constant λ becomes very small or perhaps zero. After this phase transition the universe has all the features of the standard Big Bang Model. The possibility of such phase transitions of the vacuum were first successfully introduced in connection with the unified theories of the weak and electromagnetic forces, where this mechanism proved absolutely crucial in order to explain the observed features of the electroweak interaction. The important point in the present search for a model of the origin of the universe is that we imagine that the universe started as a DeSitter space without any mass present but a large vacuum energy. Up to this point the description of the evolution of the spatial universe is entirely classical in nature with no need to incorporate quantum aspects of gravity and of space. However, in order to explain the appearance of a DeSitter space of size 10^{-26}cm at 10^{-40} sec with a large vacuum energy (of the order 10^{103} erg cm^{-3}) one must invoke quantum principles.

INFLATIONARY UNIVERSE

Expanding De Sitter Space with Phase Transition to Standard Model



$$\text{Inflation: } R(t) \approx R_i\, e^{ct/R_i}$$

FIG. 6

5. THE UNCERTAINTY PRINCIPLE OF SPACE GEOMETRY:

There is at present still no fully developed Quantum Theory of Space Geometry, no theory of quantum gravity. But we can say at what level quantum gravity effects become important and give some general features of a quantum theory of space. In the arena of present day particle physics the gravitational interaction between the particles is totally negligible compared to the interaction energies due to the other forces. We can ask: when does the gravitational energy of a mass energy package become comparable to its relativistic energy? The gravitational energy of a massive particle confined to a size ΔL is given by $GM^2/\Delta L$. So only when the mass is confined to small size will the gravitational energy become significant. Quantum mechanically a particle cannot be localized more precisely than given by its Compton ware length $\lambda_c = \hbar c/E$. Thus one finds the ratio of gravitational to total particle energy as (Fig. 7)

$$\frac{W_{grav}}{E} \approx \left[\frac{L^*}{\Delta L}\right]^2$$

where L^* is a universal length that can be formed from the three fundamental constants c, $\hbar$ and G and is of the very small size order of 10^{-33}cm. This so called Planck–Length becomes the fundamental quantum length of geometry where space quantum graininess makes its appearance. Just as $\hbar$ limits the accuracy in with which one can hope to measure the action of a process, so L^* limits the accuracy with which one can hope to determine the nature of the geometry by local measurements (Fig. 8). In order to measure and resolve spatial distances ΔL one must employ probing particles or waves with corresponding wave length of the same size ΔL. Those probing particles represent energy which in turn contributes in the same way as mass to the curvature of space and thereby distorts the

WHEN IS GRAVITATIONAL ENERGY IMPORTANT?

- Gravitational Energy of Mass M confined to size ΔL $\qquad W_{grav} \approx \frac{GM^2}{\Delta L}$
- Rest ernergy of Mass M $\qquad E = Mc^2$
- Compton Wave length of M $\qquad Mc^2 = \frac{\hbar c}{\lambda_c}$
- For Particle confined to its Compton wave length $\qquad \Delta L = \lambda_c = \frac{\hbar c}{E}$
- $$\frac{W_{grav}}{E} = \frac{GM}{c^2 \Delta L} = \frac{GE}{c^4 \Delta L} = \frac{G\hbar}{c^3}\frac{1}{\Delta L^2}$$

$$\boxed{\text{PLANCK LENGTH} \quad \left(\frac{G\hbar}{c^3}\right)^{1/2} \equiv L^* = 1.6 \times 10^{-33}\ \text{cm}}$$

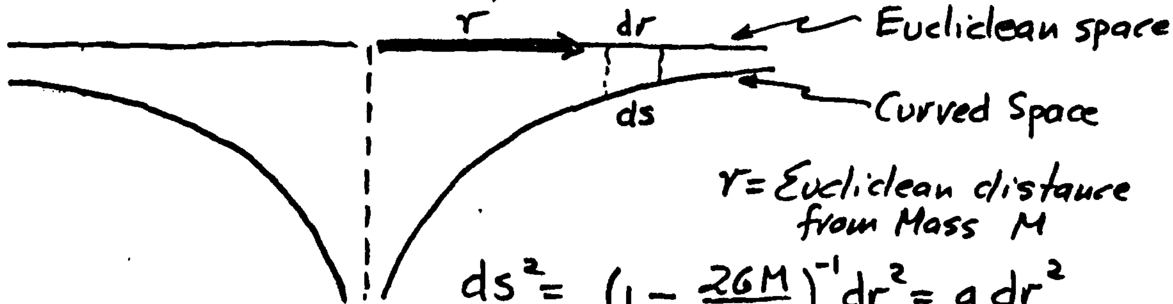
$$\frac{W_{grav}}{E} \approx \frac{L^{*2}}{\Delta L^2}$$

Only for masses confined to the order of the Planck Length L^* does the gravitational energy enter significantly

FiG. 7

UNCERTAINTY PRINCIPLE OF SPACE-GEOMETRY MEASUREMENTS

- Curved Space Geometry near mass M:



$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 \equiv g\, dr^2$$

- Newtonian Gravitational Potential near M

$$\phi_{grav} = -\frac{GM}{r} \qquad g \approx 1 + \frac{2\phi_{grav}}{c^2} \quad \text{for } r \gg \frac{2GM}{c^2}$$

- Requirement to resolve spatial distances of size ΔL:

In the wave picture the wave length of probing particle must be $\leq \Delta L$

$$\lambda_B = \frac{\hbar c}{E_{part}} \approx \Delta L$$

Energy of Probing Particle

$$E_{part} = mc^2 \approx \frac{\hbar c}{\Delta L}$$

- Gravitational Perturbation caused by Probe:

$$\Delta\phi \approx \frac{Gm}{\Delta L}$$

$$\Delta g \approx \frac{\Delta\phi}{c^2} = \frac{G}{c^2 \Delta L}\,\frac{\hbar c}{c^2 \Delta L} = \frac{L^{*2}}{\Delta L^2}$$

$$\boxed{\Delta g_{meas} \cdot \Delta L^2 \approx L^{*2}}$$

FIG. 8

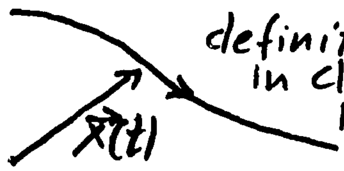
distances to be measured. The deviation of the space metric from euclidean by the presence of mass–energy is given by the change of the metric coefficients g. The above considerations lead to a uncertainty relation for the measurement of the space geometry

$$\Delta g_{measure} \cdot \Delta L^2 \approx L^{*2}$$

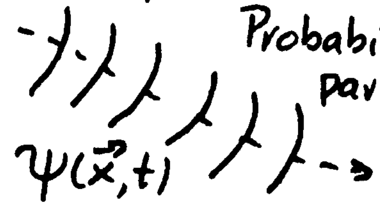
For measurements of distances of the order of L* the probing particle introduces distortions which are as large as g itself and therefore L* defines the smallest length for which geometry of space has definite meaning. A consequence of the uncertainty principle in quantum physics is that repeated measurements of the dynamic variables such as, position and momentum of a particle on its trajectory gives different answers from measurement to measurement (Fig. 9). The picture of a smooth classical trajectory is no longer meaningful and must be replaced by a smeared out wave package which represents the probability distribution of what a position measurement is likely to give. In terms of the measurement results one obtains, the particle appears to fluctuate. Measurement of energy density at a given point in space gives differing results of each measurement which we interpret by saying that the energy density at a given point cannot have a definite value (zero). Measurements give fluctuating values around zero and we must accept this as fact and say: the energy density is fluctuating in our reality. Similarly the uncertainty principle of geometry forces us to assume that the dynamic variable under consideration, the geometry of space cannot have the definite euclidean form. It must show fluctuations which appear as a turbulent random fluctuation away from euclidicidy. Local curvature of space is caused by the presence of mass–energy and so the metric fluctuation can be thought of as caused by fluctuation of local energy density, and hence the appearance and disappearance

"VACUUM-FLUCTUATION" OF THE SPACE METRIC FIELD

- Uncertainty in Mechanics and Geometry:



definite Trajectory in classical Mechanics



Probability of particle position

Definite Locally Euclidean Geometry

Fluctuating Geometry due to Quantum Uncertainty

- Wavelength associated with energy fluctuation $\Delta E \approx \Delta m\, c^2$: $\Delta L \approx \dfrac{\hbar c}{\Delta E}$

- Corresponding Gravitational Potential field and gravitational energy density

$$\Delta\phi \approx \frac{G\,\Delta m}{\Delta L} \qquad \Delta W_{grav} \approx \frac{G\,\Delta m^2}{\Delta L}$$

$$\Delta g_{fluct} \approx \frac{\Delta\phi}{c^2}$$

- Gravitational Distortion becomes important when $\Delta E \approx \Delta W_{grav}$

$$\boxed{\Delta g_{fluct} \cdot \Delta L \approx L^*}$$

FIG. 9

of mass. Associated with a mass energy fluctuation of $\Delta\epsilon \approx \Delta mc^2$ is its corresponding Compton's wave length

$$\Delta L \approx \hbar c/\Delta\epsilon$$

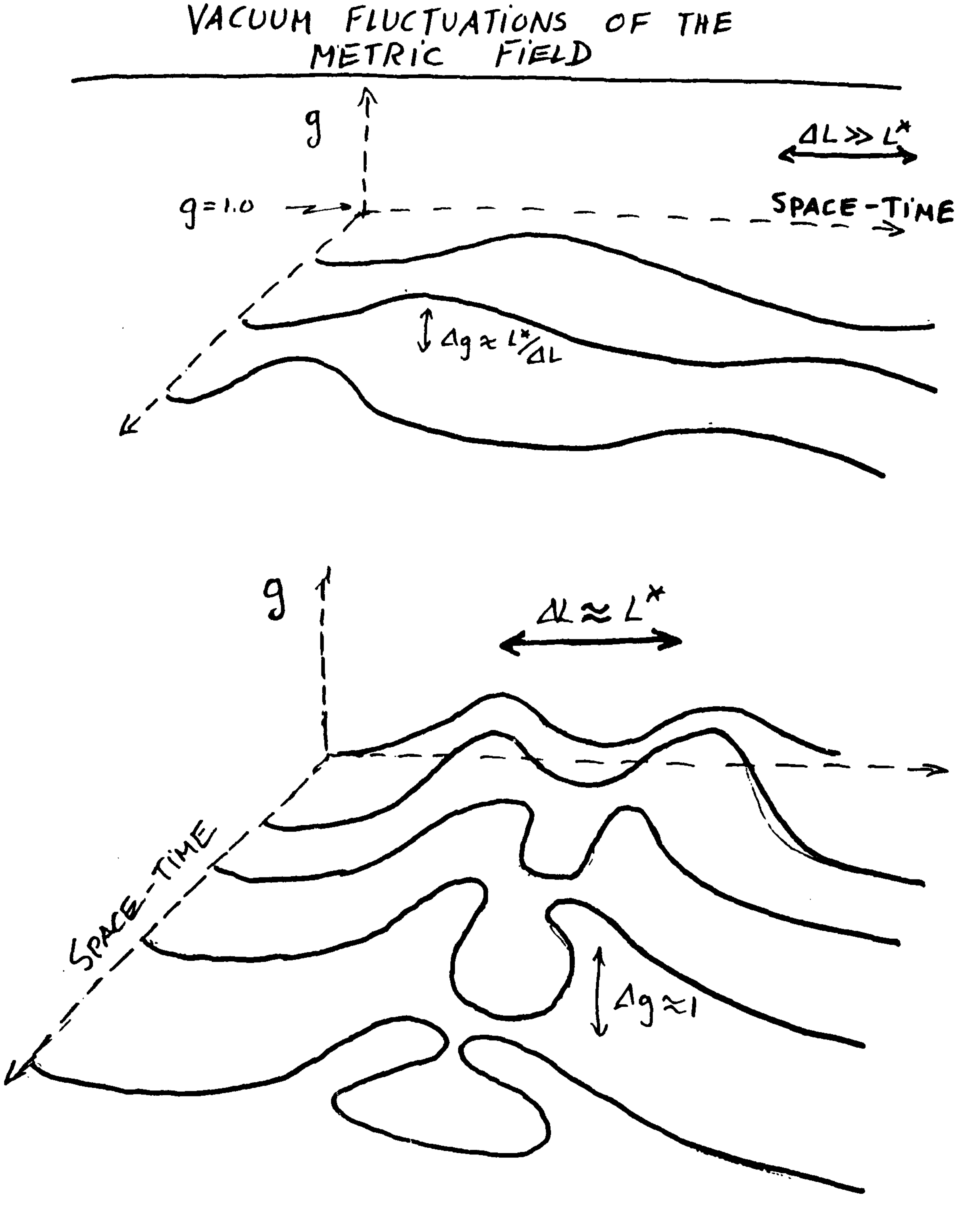
Over the distance ΔL the mass Δm will cause a gravitational potential $\Delta\phi$ and a corresponding metric fluctuation Δg_{fluct}. Most energy fluctuations are of wave lengths that do not cause significant gravitational effects. Only when the gravitational energy of the fluctuation is comparable to the energy itself are the influences significant. This leads to

$$\Delta g_{fluct} \, \Delta L \approx L^*$$

which says, that if we are considering geometric features of distances large compared to L^* we will find the metric fluctuations to be insignificant (Fig. 10). But over distances of order L^* we will see significant fluctuation. The picture is somewhat like observing the ocean from high altitude when it appears as a very smooth surface: however if one is sitting in a small boat observing the surface at close range one may be overwhelmed by the warped structure of this surface, by the energy involved in the processes of these fluctuations and by the fact that the surface may not even be continuous but grainy due to breaking wave crests. The energy density of these vacuum fluctuations is of the order:

$$\frac{\Delta\epsilon}{L^{*3}} \approx \frac{\hbar c}{L^{*4}} \approx 10^{115}\ \mathrm{erg cm^{-3}}$$

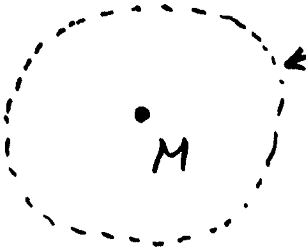
A first application of the uncertainty principle of gravitation which was made by Stephen Hawking was the discovery that Black holes cannot be totally black. Classically, a mass M



VACUUM FLUCTUATIONS OF THE METRIC FIELD
g
g=1.0
ΔL ≫ L*
SPACE-TIME
Δg ≈ L*/ΔL
g
ΔL ≈ L*
SPACE-TIME
Δg ≈ 1

FIG. 10

BLACK HOLE EVAPORATION AS QUANTUM TUNNELING



Schwarzschild Radius $R_c = \dfrac{2GM}{c^2}$

- The total energy of the system confined to R_c is uncertain by quantum effects. The amount ΔE can escape through the Schwarzschild surface if the uncertainty persists a time $\Delta t \approx R_c/c$

- Uncertainty Principle: $\Delta E \sim \dfrac{\hbar}{\Delta t}$

$$\Delta E \approx \frac{\hbar c}{R_c} \approx \frac{\hbar c^3}{2GM}$$

- Power Radiated:

$$P = \frac{\Delta E}{\Delta t} = \frac{\hbar}{\Delta t^2} = \frac{\hbar c^2}{R_c^2} = \frac{\hbar c^6}{4G^2M^2}$$

$$= \left(\frac{\hbar c}{G}\right)\left(\frac{\hbar c}{G}\right)^{1/2}\left(\frac{c^3}{\hbar G}\right)^{1/2}\frac{c^3}{4M^2}$$

$$= \frac{1}{4}\,\frac{c^2}{t_P}\,\frac{M_P^3}{M^2} \qquad = -\frac{d(Mc^2)}{dt} \cong 10^{-50} L_\odot \left(\frac{M_\odot}{M}\right)^2$$

- Lifetime of Black Hole:

$$\tau_H \sim \frac{4}{3}\left(\frac{M}{M_P}\right)^3 t_P \approx 10^{66}\left(\frac{M}{M_\odot}\right)^3 \text{ years}$$

- $M_P \equiv \left(\dfrac{\hbar c}{G}\right)^{1/2} = 10^{-5}$ gr $\qquad t_P = \dfrac{1}{c}\left(\dfrac{\hbar G}{c^3}\right)^{1/2} \doteq 10^{-44}$ sec.

FIG. 11

that has collapsed within its Schwarzschild radius $R_c = 2GM/c^2$ will not allow any signal to climb out of this deep potential well. From a quantum point of view the confinement of energy Mc^2 within a finite well is associated with some energy uncertainty

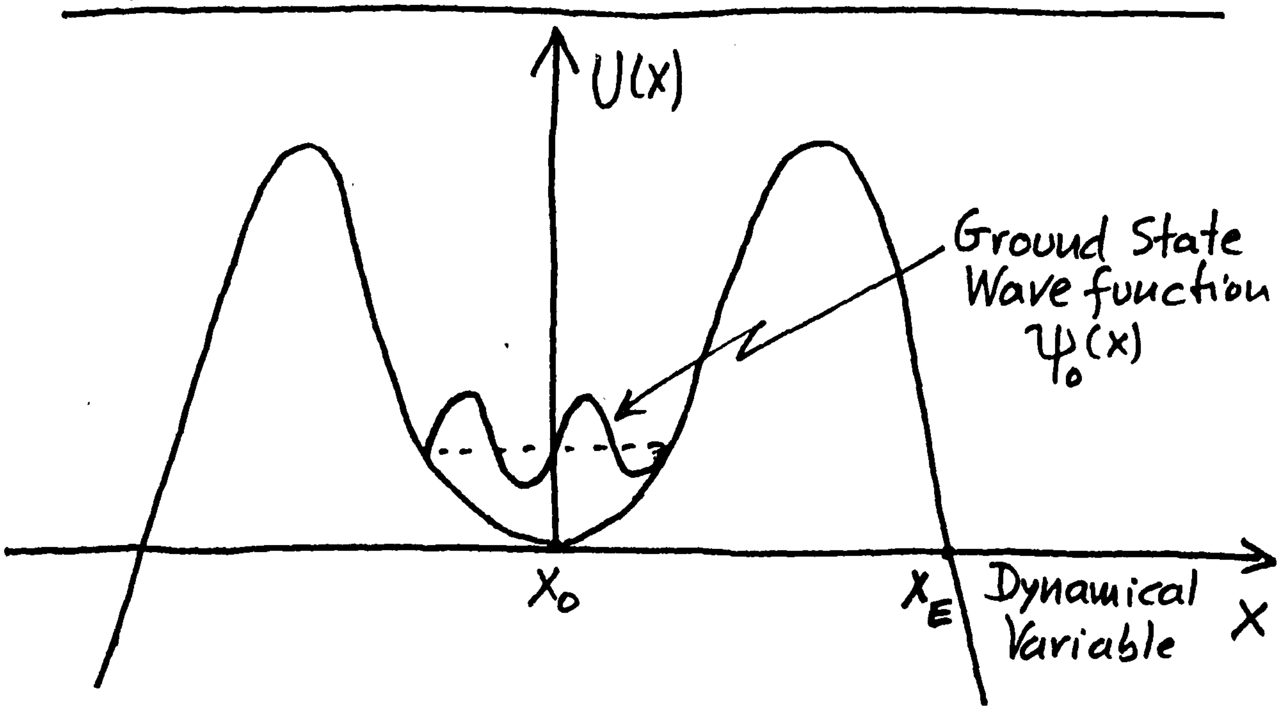
$$\Delta E \sim \frac{\hbar c}{R_c}$$

If this energy uncertainty exists long enough to allow transport across the distance R_c then there is a finite probability for energy to actually "tunnel" out of the potential well. The power radiated this way is $\Delta E/\Delta t$ with $\Delta t \approx R_c/c$. One would expect the remnants of super novae which may have collapsed to become Black holes, to have a mass on the order of one solar mass. Far masses of that order one finds the radiated power to be very small. The lifetime of such Black Holes due to this "Hawking – Radiation" would be of the order of 10^{66} years. As the hole looses mass very slowly it becomes ever more easy for energy to tunnel out. The last 10,000 tons of material evaporate with a flash of luminosity equal to that of the sun within 1/100 second. It is this phenomenon of quantum tunneling that can be viewed as the mechanism leading to the birth of our universe as well.

6. QUANTUM TUNNELING FROM POTENTIAL WELLS:

Consider a mass point in a potential well and having zero total energy for the system. Classical mechanics leads to the unique result that the particle is stationary with definite momentum equal to zero at the equilibrium point of the potential well. If one considers the problem quantum mechanically then the particle cannot have a definite energy, momentum and position according to the uncertainty principle. The particle's energy fluctuates as well as its momentum and position. The behavior of the particle is described by a wave function $\psi_0(x)$ for the "ground state" of the system which gives the probability

QUANTUM MECHANICS OF POTENTIAL WELL



Classical Mechanics $\Longrightarrow$ Quantum Mechanics

For $E=0$

$$\frac{P_x^2}{2m} + U(X) = 0 \qquad \Longrightarrow \left\{\frac{1}{2m}\left(-\frac{i}{\hbar}\frac{\partial}{\partial x}\right)^2 + U(X)\right\}\psi(x) = 0$$

Ground state:

$P_x = 0$

$X = X_0$

$\psi_0(x)$

Due to Uncertainty Principle the system cannot be at exactly $x = x_0$ with exactly $E=0$

FIG. 12

distribution for the location of the particle which fluctuates in the well. If the potential well is shaped as in Fig. 12 with regions outside where the potential is actually lower than at the equilibrium point in the well, then there is a finite probability that the particle will tunnel out of the well or, saying it another way, due to the energy uncertainty there is the possibility of energy excursions large enough to climb over the barrier. The ground state wave function is a solution to the Schrödinger equation of the problem for energy eigemvalve value zero. The laws of quantum mechanics actually allow one to calculate the probability for the particle to tunnel through the barrier which in the picture of wave mechanics is like the refraction of a wave through a medium in which the refractive index is imaginary. This probability is given by the approximation:

$$\Gamma \approx A \exp \left[-\frac{2}{\hbar} \int_{X_0}^{X_E} \sqrt{2U(x)}\, dx\right]$$

Classically the region where U(x)>0, is forbidden, since the energy equation

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + U(x) = 0$$

gives imaginary velocities in that region. Suppose one looks at an associated motion problem: that of the same particle of unit mass in the potential which is the negative of U(x) (Fig. 13). The particle with zero total energy starting at X_0 will begin infinitely slowly to roll down the potential well, climb up the other side, reach the point X_E reverse direction and roll back and after infinite time reach X_0 again. This motion is called a "bounce" and it satisfies the energy equation

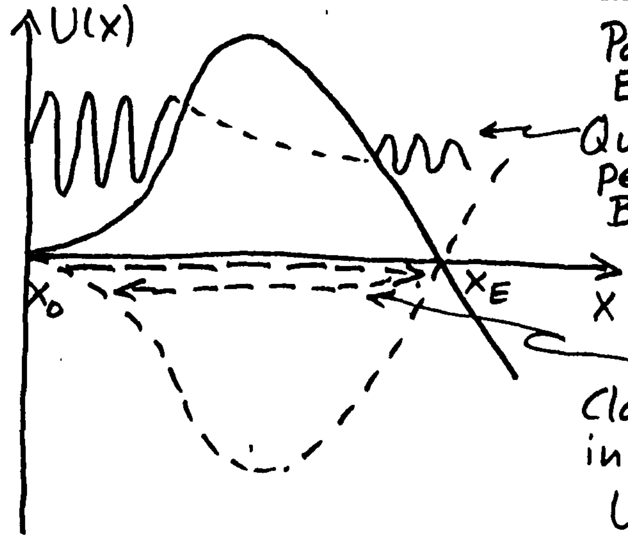
$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 - U(x) = 0$$

The same form of the energy equation with the minus sign can be obtained from the original energy equation with the plus sign by transforming time t to an imaginary time $t \to i\tau$. In other words, if one analytically continues the energy equation

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + U(x) = 0$$

to imaginary time one obtains as solutions the bounce motion. If one calculates the total action for the motion with imaginary time one obtains exactly the exponent in the expression for the probability of barrier penetration. So one can then treat the entire motion of a particle tunneling through the potential barrier, its motion outside the barrier in real time after the penetration and its motion through the barrier by analytic continuation to imaginary time with the motion starting in the infinite imaginary time past. In a $X - \tau$ diagram the bounce motion starts at $\tau = -\infty$ at $X = 0$, reaches a maximum at X_E at $\tau = 0$ and returns to $x = 0$ at $\tau = +\infty$. The classical motion outside the potential well is spliced in real time to the bounce motion at $\tau = 0$ at X_E. The trajectory of the bounce in the $X - \tau$ plane is a single hump centered at $\tau = 0$ (Fig. 14). It is a stationary "solution" in imaginary time. Such solutions are called "Instantons". The purpose of this exercise is that it allows to treat the barrier penetration and subsequent external motion in a unified classical way and to allow to estimate certain quantities which usually require a full quantum mechanical treatment. The complete motion is spread over infinite, albeit imaginary time and does not require any particular initial condition to define the solution. These principles are applicable to what one might call Quantum Cosmology.

QUANTUM TUNNELING THROUGH POTENTIAL BARRIER



Particle of mass $m = 1$
$E = 0$

Quantum Mechanic Barrier penetration through Barrier $U(x)$

Classical "Bounce"-Motion in Potential well
$$U'(x) = -U(x)$$
$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 - U(x) = 0$$

- Probability of Barrier Penetration:
$$T = A \exp\left[-\frac{2}{\hbar}\int_{x_0}^{x_E}\sqrt{2U(x)}\,dx\right] = A\exp\left[-B/\hbar\right]$$

- Classical Bounce Motion as Motion in imaginary time:
$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + U(x) \xRightarrow{t = i\tau} \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 - U(x) = 0$$

  Barrier Penetration can be classically described as Bounce Motion in imaginary time

- "Euclidean" Action of Bounce Motion:
$$S_E = 2\int_{x_0}^{x_E}\left(\frac{dx}{d\tau}\right)dx = 2\int_{x_0}^{x_E}\sqrt{2U(x)}\,dx \equiv B$$

FIG. 13

"INSTANTONS"

Stationary Soliton in Imaginary Time

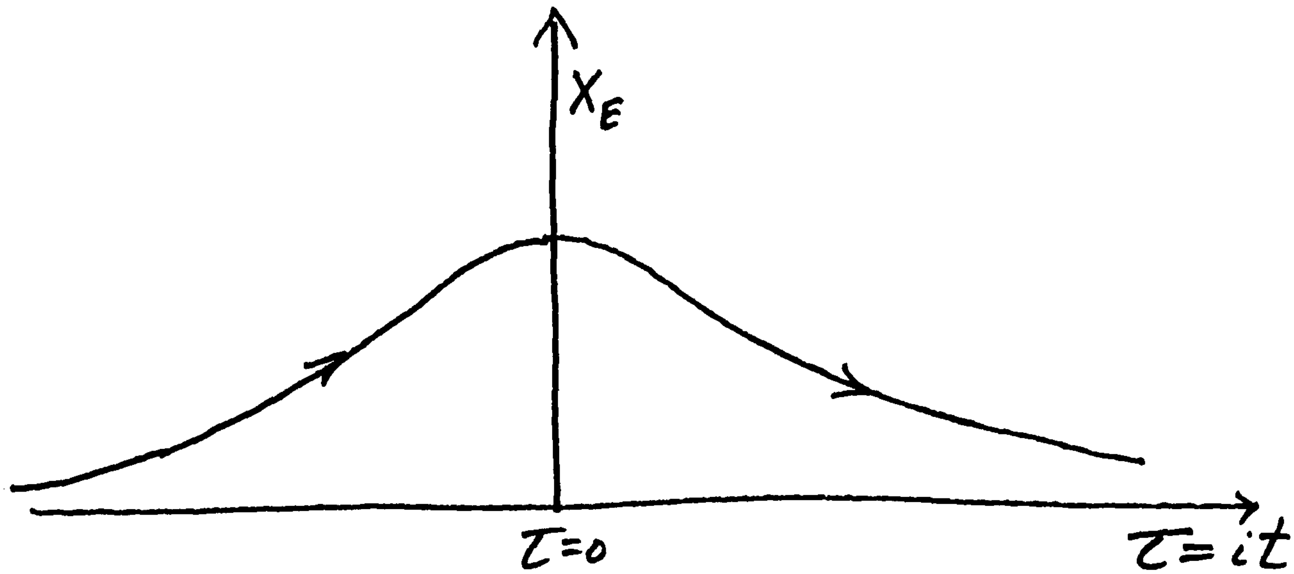


Fig 14

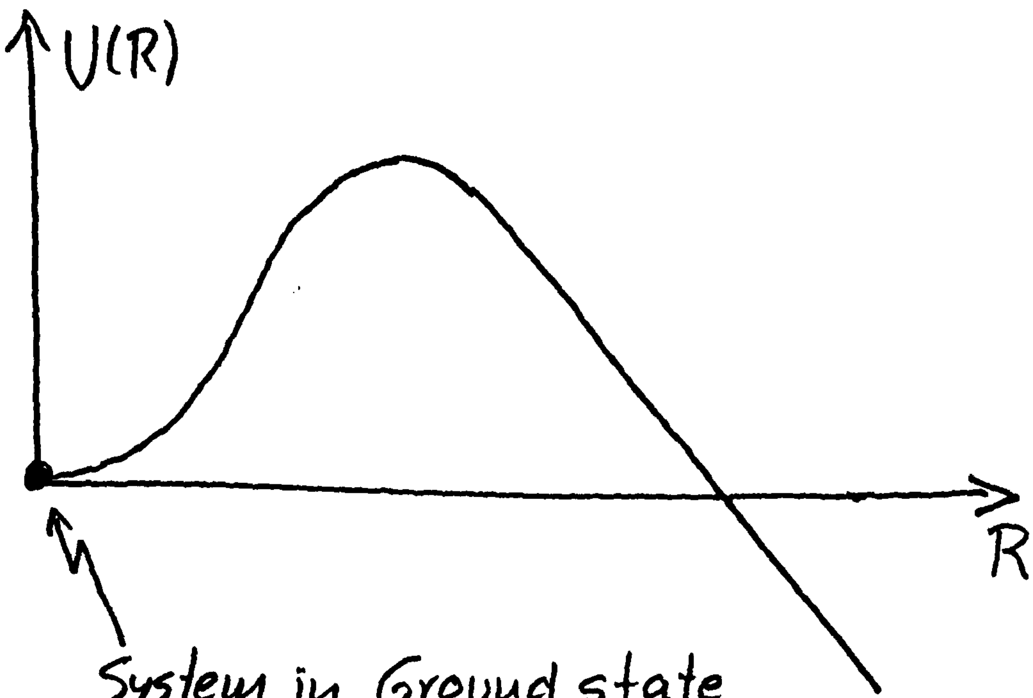
7. QUANTUM COSMOLOGY

In the quantum mechanics of a particle in a potential well the dynamical variable is the position of the particle. The system in the ground state is represented by a ground state wave function. The particle fluctuates around the equilibrium point $X = 0$ and eventually tunnels out of the well, appears at X_E and moves outward from there. In Cosmology the dynamical variable is the radius of the universe R (Fig.15). Quantum mechanically the universe cannot have a exact definitive value for R and in particular not $R = 0$. The universe must be described by some wave function $\psi(R)$ which gives the probability of finding the universe with radius R. The wave function satisfies a kind of Schrödinger equation, which is named the Wheeler – DeWitt – Equation after their inventors and which contains some potential function U(R). All the physics of the universe is contained in this potential function just as all physics of the hydrogen atom is contained in the Coulomb potential. In order to describe the appearance of an expanding DeSitter space that potential must have some form like our potential well in the mechanics analog. In the "ground state" the radius of the universe is classically zero, i.e. no extended space exists at all. Quantum cosmologically the state of the system is described by a ground state wave function $\psi_0(R)$ which is not zero. It gives the probability of finding rudimentary universes of radius R inside the well. By quantum fluctuation little bubble universes are constantly forming and disappearing. The ground state of "world" is a infinity of bubbles of size of order 10^{-33} cm changing like a bubbling foam with time scales of 10^{-44} sec (Fig. 16). No measurable extended space exists. But there is a finite probability that one bubble fluctuation is large enough, say 10^{-26} cm, that the system that was stuck in the ground state, spontaneously tunnels through the potential barrier. Now, it is a classical DeSitter space which due to the decreasing potential rapidly expands to

QUANTUM COSMOLGY

- Dynamic Variable : R = Radius of De Sitter Space
- Wave function for Universe: $\Psi(R)$
- "Schrödinger Equation of Quantum Cosmology (Wheeler - De Witt - Equ)

$$\left[\left(\frac{\partial}{\partial R}\right)^2 - U(R)\right]\Psi(R) = 0$$



System in Ground state

classicaly: Dynamical Variable R = 0 (no space)

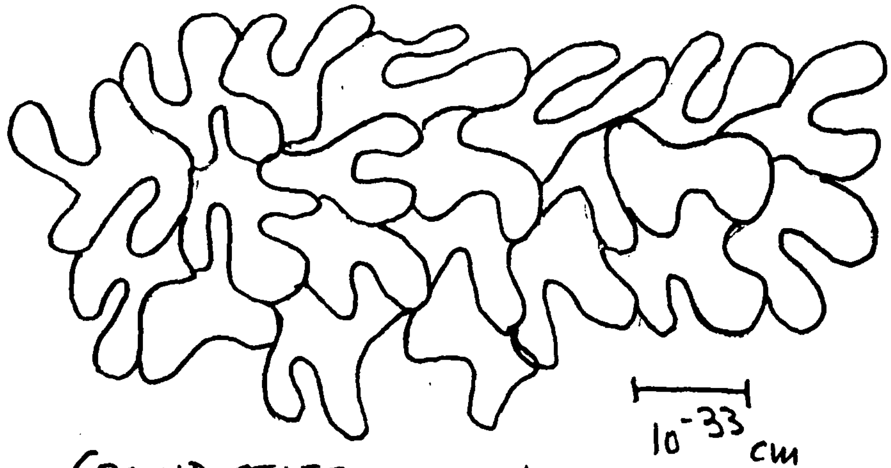
quantum Mechanically:

Uncertainty Principle

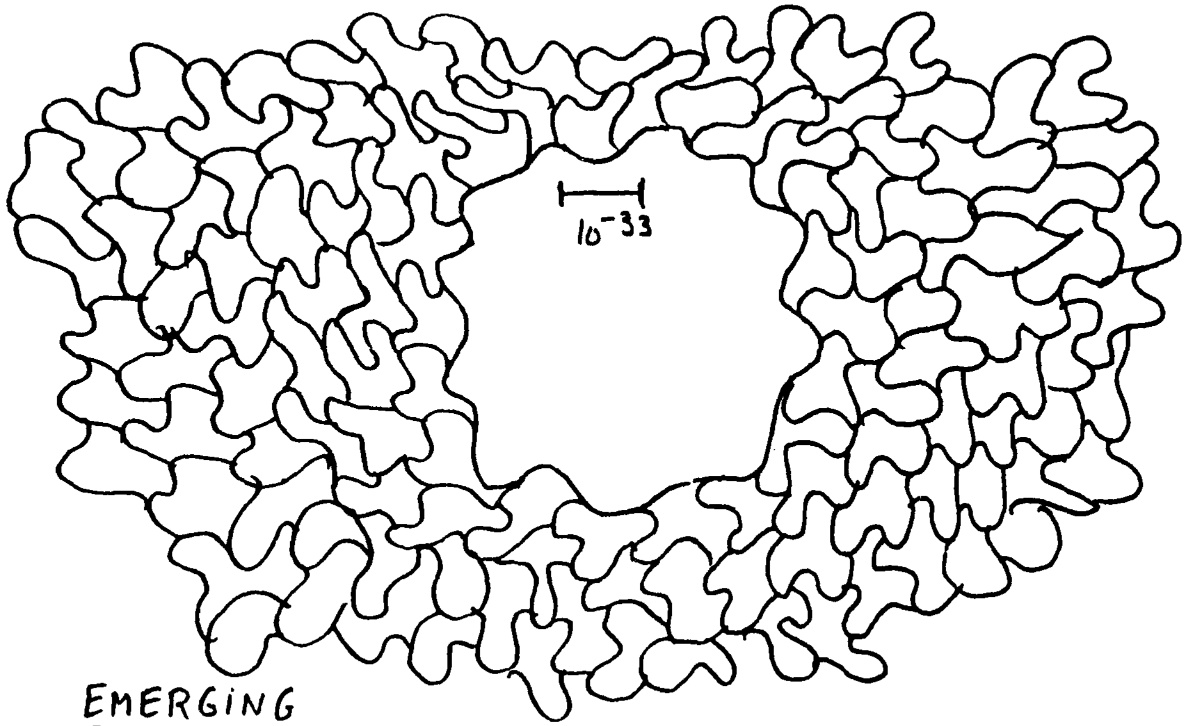
Quantum Chaos of Space Fluctuations

Groundstate wave function $\Psi_0(R)$

FiG. 15



GROUND STATE OF UNIVERSE:
Chaotic Fluctuations of Spatial Geometries according to Quantum Uncertainty principle on time scales of $\Delta t \approx 10^{-44}$ sec



EMERGING INFLATIONARY De Sitter SPACE

$R(t_i) \approx 10^{-26}$ cm

FIG. 16

large valves of R. Prior to the tunneling event only short range fluctuations of dimension 10^{-33}cm were present. After the tunneling a "long wave" structure makes its appearance on the background of the quantum fluctuations. The energy density that these quantum fluctuations represent is of the order (Fig. 17).

$$u \approx \frac{\hbar c}{L^{*4}} \approx 10^{115} \text{ ergcm}^{-3}$$

The newly born long range structure must lay some order over this fluctuating background, it must so to speak climb over the entire energy collected in these fluctuations (Fig. 18). That allows us to make an estimate of the magnitude of the potential barrier that the system has to penetrate in order to emerge as a classical DeSitter space of initial radius Ri. We can estimate that the barrier should be of height.

$$U_m \approx Ri^3 \frac{\hbar c}{L^{*4}}$$

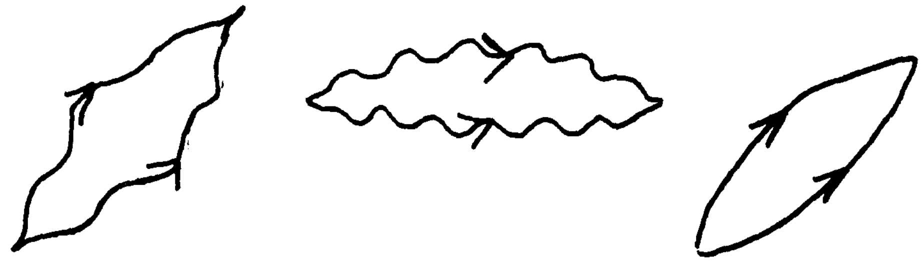
Now we can estimate the tunneling probability by estimating the action of the corresponding bounce motion in the inverted potential

$$\Delta S \approx U_m \cdot \frac{Ri}{c}$$

where Ri/c is the time required for the bounce motion. The result is

$$\Delta S \approx \hbar \left[\frac{Ri}{L^*}\right]^4$$

ENERGY DENSITY OF VACUUM FLUCTUATIONS



VACUUM ENERGY FLUCTUATIONS $\Delta E \cdot \Delta t \approx \hbar$ $\longleftrightarrow$ Pair creation and Annihilation $\longleftrightarrow$ Zero Point Energy of Sea of Quantum Harmonic Oscillators

Total Energy $U = \sum_{\substack{\text{all}\\ \text{oscillators}}} \sum_{\substack{k\\ \text{all modes}}} \frac{1}{2}\omega_k \hbar$

For Continuum:

$$U = \int_{L^3} d^3x \int \frac{1}{2}\omega_k \hbar \, d^3k$$

With $\omega_k = ck \qquad d^3k \approx k^2 dk$

$$U \approx L^3 c\hbar \int_{1/L}^{1/L^*} k^3 dk$$

Wave number limits $\frac{1}{L} \leq k \leq \frac{1}{L^*} \qquad L^* = 10^{-33}$ cm

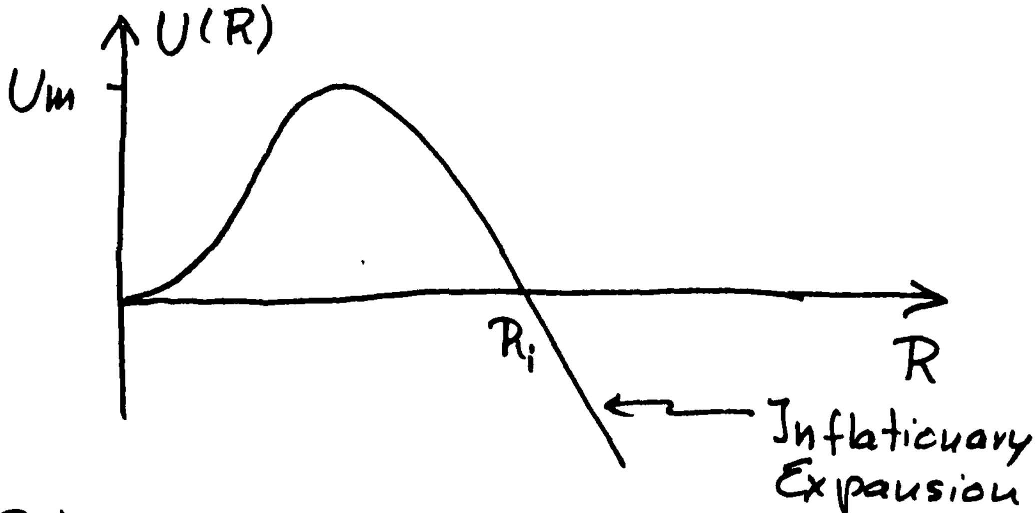
For $L >> L^*$

$$U \approx \frac{\hbar c}{L^{*4}} L^3$$

Energy density $u \approx \frac{\hbar c}{L^{*4}} = 10^{115}$ erg cm^{-3}

FIG. 17

ESTIMATE OF TUNNELING PROBABILITY



- Potential Maximum: U_m
  characterized by Energy density of "Vacuum" Fluctuations

$$u \approx \frac{\hbar c}{L^{*4}} \approx 10^{115} \text{ erg cm}^{-3}$$

$$U_m \approx R_i^3 \cdot \frac{\hbar c}{L^{*4}}$$

- Action of the corresponding classical Bounce motion:

$$\Delta S \approx U_m \frac{R_i}{c} = \hbar \left(\frac{R_i}{L^*}\right)^4$$

For $R_i \approx 10^{-26}$ cm $\quad L^* = 10^{-33}$ cm

$$\frac{\Delta S}{\hbar} \approx 10^{28}$$

Tunneling Probability $\quad T \approx e^{-10^{28}}$

FIG. 18

With $Ri \approx 10^{-26}$cm one finds a tunneling probability of

$$\Gamma \approx e^{-10^{28}}$$

This is an unfathomable small number. How small it is one can perhaps see if one calculates the total number of possible events in our present universe during the time of its existence. The present radius is $R_0 \approx 10^{28}$cm. This universe contains $R_0{}^3/L^{*3} \approx 10^{183}$ cells of size 10^{-33}cm. These cells fluctuate with time scales of 10^{-44}sec which gives the highest rate at which anything meaningful can happen. The present age of the universe is 10^{17}sec so there are 10^{61} time chunks of 10^{-44}sec. The total number of possibly meaningful events is then 10^{245}. Each of these events is one little bubble lifetime that could have grown by tunneling into a full universe. So the probability of forming a universe in a region of size of present universe over a time period of the age of the universe is

$$\text{Prob} \approx 10^{245} \cdot e^{-10^{28}}$$

The factor 10^{245} makes no dent in the factor $e^{-10^{28}}$ and we may have a glimpse of eternity and infinity here.

8. CONCLUSION

The numbers derived above are highly speculative. The tunneling probability depends crucially on the potential U(R). Physics today is far from having a correct Quantum Cosmology and a theory of "everything". The potential U(R) contains only one dynamical variable, which is associated with a very special class of possible universes. In order to describe the world as we know it, in which other forces besides gravity must be

HOW SMALL is $e^{-10^{28}}$?

Radius of present universe	$R \approx 10^{28}$ cm
Volume of universe	10^{84} cm^3
Number of cells of size L^* in universe	$\frac{10^{84}}{(10^{-33})^3} \approx 10^{183}$
Age of universe	10^{17} sec
Number of elementary fluctuation of duration $L^*/c \approx 10^{-44}$ sec	10^{61}
Total number of "chances" to start a universe in Volume of size of presend universe in time of the age of universe	10^{245}
Probability that universe could have happened in Volume of present universe In time since birth of this universe	$10^{245} \cdot 10^{-0.4 \times 10^{28}}$

FIG. 19

incorporated and in which matter as we know it makes its appearance after some phase transition in the inflationary DeSitter space, it now appears that the correct theory of "everything" may be some form of 10–dimensional super string theory. In that case there would be many more dynamical variables describing the universe depending on the degree of symmetry. All of these variables could enter into the potential function which could be a very complicated hypersurface over a multidimensional space. If the chaotic ground state consists of quantum fluctuations in a 10–dimensional space, then the theory must somehow describe why this chaos tunnels to a state in which 3 dimensions undergo inflationary expansion with a large vacuum energy that eventually can condense out to become matter and why other six space dimensions somehow curl up into an organized foam not visible even on microscopic scales accessible to us, which however manifests itself as the other forces in nature. It must also explain why the universe is born with a large cosmological constant that has the tendency to go to zero at the phase transition of the DeSitter space. All this information must be contained in the potential U(R,...). Even the whole scenario described here: quantum tunneling from the "nothingness" of quantum chaos to a classical DeSitter space with no matter but high vacuum energy (cosmological constant) and subsequent phase transition to the standard Big Bang model with matter and zero cosmological term, is by no means certain. All these details of the actual birth process of our universe cannot be narrowed down before one has a "theory of everything" that gives some semblance of the actual observed behavior of space and matter at lower energies. Nevertheless, the scenario described here shows that it is now possible to bring together known principles of physics and apply them to construct models of the origin of the universe that are more than mythological images, and have the beginnings of some quantitative features.

Inquiry into the origin of the universe brings most people into touch with the other "ultimate questions of being": the search for the Creator and the search for the meaning of our own existence. It should be clear that whatever can be said about the origin of the universe contributes nothing to the question about the nature of the Creator and the act of Creation. The equations of quantum cosmology may or may not describe what actually happened. Eventually we may even arrive at the nearly correct model. These models and equations can be generated in our heads. But no act on our part can convert such thoughts into actuality, real being. Yet the universe which we explore appears to be real. By some act or mechanism the being of this universe is actualized, the thoughts contained in the equations are rendered manifest in real beingness. This mechanism or act is outside the avenues of inquiry described here and belongs rightfully into the realm of metaphysics, of that which is beyond the physical universe of which we are but a small part. These theories still can only claim to describe what happened and how it happened. It leaves the deeper question why there is a World and not Nothingness untouched. If anything, these explorations on the origin of the universe may help to see that the fundamental questions about Being and the Creator or creating mechanism for Being must operate on an entirely different level, which can only fill us with awe.

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